USEFUL WORK AND INFORMATION AS DRIVERS OF GROWTH

Robert U. Ayres

Benjamin Warr*

CMER

INSEAD

* **Robert U. Ayres and Benjamin Warr**

Center for the Management of Environmental Resources

INSEAD

Boulevard de Constance

77305 Fontainebleau Cedex

France

robert.ayres@insead.edu

benjamin.warr@insead.edu

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Abstract

The history of growth theories is reviewed. A semi-empirical endogenous growth theory is proposed. It is based on a model of the economy as a multi-stage materials processing system. Growth is simulated by a two-parameter production function with two traditional factors, labor and capital, and a non-traditional factor. The non-traditional factor can be interpreted as `useful' (physical) work output, as distinguished from energy (exergy) inputs. It is calculated from energy (exergy) inputs multiplied by the average energy conversion efficiency, which is a function of time. This theory `explains' past US growth from 1900 through 1973-74 with satisfactory accuracy but it slightly underestimates subsequent growth (i.e.it leaves a small unexplained residual) for the period after 1975. However, by subdividing capital stock (and services) into traditional and information-technology components, we are able to extend the theory to explain US economic growth accurately through 1998. The revised production function has only three independent parameters, determined by statistical fitting. The new theory also has implications for future economic growth, energy and environmental policy that differ significantly from the traditional growth theory. These implications are discussed very briefly.

Background

Most economic theory since Adam Smith has assumed the existence of an equilibrium between supply and demand; it is this equilibrium that permits the beneficent functioning of the `invisible hand'. The notion was successively refined by Ricardo, Say, Walras, Jevons, Wicksell, Edgeworth, Pareto and others in the 19th century. Walras postulated a *competitive equilibrium* such that supply and demand is always satisfied, at stable prices, and all markets (including labor markets) clear. He also postulated a sort of auction process, known as *tâtonnement*, by means of which prices are determined in a public manner without individual pairwise bargaining. Walras' postulate was widely accepted, though not proved (Kenneth J.

Arrow and Gerard Debreu, 1954, Abraham Wald, 1936) until long after his death. Since then most economists have assumed that the real economy is always in, or very close to, Walrasian equilibrium (Robert M. Solow, 1970).

The Walrasian model was static. It applied only to exchange transactions, and did not attempt to explain growth. Growth was attributed in the 19th century to labor force growth and capital accumulation. In the 18th century, when capital primarily meant land, and when most arable land in Europe was already being tilled, it was not clear how a growing population could be fed from a finite supply of land. This was the conundrum that motivated Thomas Malthus to write his pessimistic assessment of the consequences of population growth in 1798 (Thomas Robert Malthus, 1946).

The role of land gradually receded in comparison with the role of industrial capital, as the 19th century went on. By the early 20th century, growth per capita was supposed to be driven by the accumulation of industrial capital. The most influential models of the 1930s and `40s were based on a formula attributed to Fel'dman (G. A. Fel'dman, 1928) equating the rate of growth of the economy to the savings rate divided by the capital-output ratio, or (equivalently) the ratio of annual savings to capital stock. The formula was rediscovered by Roy Harrod in 1939 (Roy F. Harrod, 1939)and by Evsey Domar in 1946 (Evsey D. Domar, 1946). These models, which emphasized the role of central planning, dominated postwar development economics thinking.

The situation changed in the early 1950s when empirical research drastically reduced the apparent role of capital (Moses Abramovitz, 1956, Solomon Fabricant, 1954, John W. Kendrick, 1956). For example, Fabricant's estimate was that capital accumulation only accounted for 10% of US economic growth since the middle of the 19th century.

The next step, in the 1950s was the explicit introduction of aggregate production functions in which capital services are derived from an artifact called `capital stock' which, in turn, is an accumulation based on investment and depreciation (Robert M. Solow, 1956, 1957), Swan (Trevor Swan, 1956). The unexplained `Solow residual' still accounted for most of the per capita growth in output. Solow named this residual `technological progress' and that identification seems to have satisfied most economists until recently. Its annual increments are called increases in "total factor productivity", and productivity calculations have become a mini-industry. However, naming a disease is not the same as explaining it.

Apart from a number of other questionable simplifications, the standard theory suffers from a crucial deficiency: it does not actually explain economic growth.

The so-called Solow residual (or technological progress) remains to be explained. The neo-classical paradigm as articulated by Solow and others does not allow for `real' material flows. Production and consumption are abstractions, linked only by money flows, payments for labor, payments for products and services, savings and investment. These abstract flows are governed only by equilibrium-seeking market forces (the "invisible hand"). There is no deep fundamental connection between the physical world and the economy. The equilibrium assumption is needed mainly to justify the assumption that output is a function of capital and labor inputs and that the output elasticities of the factors of production (i.e. marginal productivities) should correspond to payments in the National Accounts.[1] This `requirement' is a consequence of the theory of income allocation between factors (capital and labor) in a population of perfectly competitive producers of a single all-purpose good, in equilibrium. Luckily, in a multi-sector model with a sequential structure, it can be shown that there is no correspondence of payment shares in the National Accounts and factor productivities if a third factor is introduced (Robert U. Ayres, 2001).

The production function approach is generally coupled with an assumption of `constant returns to scale'which essentially means that N copies of an economic system would produce exactly N times the output of one system. This assumption is plausible, at least for a large system. It is mathematically very convenient, since it sharply limits the mathematical forms of allowable production functions to homogeneous functions of the first order, also known as the Euler condition.

There are a number of other serious problems with the neoclassical growth-in-equilibrium assumption. While it is possible to generate a sort of growth process in a single sector general equilibrium model by postulating an exogenous growth driver in the form of a smoothly increasing multi-factor productivity multiplier (Robert M. Solow, 1956, 1957, John von Neumann, 1945), this is not consistent with observed patterns of structural change that would have to be reflected in multi-sector models. Detailed critiques of the equilibrium assumption exist, e.g. (Niko Kaldor, 1985, Janos Kornai, 1973). A point that seems crucial to us is that Walrasian equilibrium is clearly inconsistent with innovation at the micro-scale or structural change at the macro-scale. Thus growth-in-equilibrium is essentially a self contradiction.

The neoclassical model has been criticized in other fundamental ways. For instance, Herbert Simon emphasized the impossibility of optimization in the absence of perfect information, proposing an alternative rule of bounded rationality and `satisficing' (Herbert A. Simon, 1955, 1959). Regrettably, most theoreticians are still devoted to formal optimization, because it allows the use of elegant Hamiltonian formalisms borrowed from classical mechanics.

Recently some theorists have experimented with the idea that growth-in-equilibrium may be explained by allowing positive returns to scale, justified by the notion that knowledge accumulated in one field of production can have spillover effects in other fields. Paul Romer postulated a tradeoff between current consumption and investment in `knowledge', which he assumes could be monopolized long enough to be profitable to the discoverer, but yet almost immediately becomes available as a free good (spillover) accessible to others (Paul M. Romer, 1986). A number of other models have been introduced, exhibiting a variety of variants on the idea. For instance, Robert Lucas (Robert E. Lucas, Jr., 1988), building on some ideas of Uzawa (H. Uzawa, 1965), focuses on `social learning' and the the tradeoff between consumption and the development of `human capital'. In the Lucas version the spillover is indirect: the more human capital the society possesses, the more productive its individual members will be. This externality is embedded in the production function itself, rather than in the knowledge variable.

Some models revert to the older Harrod-Domar(Evsey D. Domar, 1946, 1957, Roy F. Harrod, 1939, 1948) AK formalism. This is done by assuming that all input factors are accumulable, hence can be considered as `capital' of some kind. One version allows two kinds of capital, `real' and human (R. G. King and S. Rebelo, 1990). An alternative version assumes one kind of capital but two sectors, one of which produces only capital from itself (S. Rebelo, 1991). Another approach was to allow non-constant returns bounded from below by preserving the distinction between accumulable and non-accumulable factors (e.g. labor, land) and modifying the production function to prevent capital productivity from vanishing even with an infinite capital/labor ratio e.g.(L. Jones and R. Manuelli, 1990).

The development of endogenous growth theory along neo-classical lines seems to have culminated, for the present, with the work of Aghion and Howitt (Philippe Aghion and

Peter Howitt, 1998, 1992) and Barro and Sala-i-Martin (Robert J. Barro and Xavier Sala-I-Martin, 1995). These authors (like Romer) focus on investment in knowledge itself (education, R&D) as a core concept. However, all of the so-called endogenous growth models share a fundamental drawback: they are essentially theoretical because none of the proposed choices of core variables (knowledge, human capital, etc.) is readily quantified, and while R&D investment is quantifiable, it does not explain economic growth.

An evolutionary theory of growth

The evolutionary approach emerged as a distinct branch of economic theory in the 1980s, although it was inspired by Schumpeter's early work (Joseph A. Schumpeter, 1912, 1934). In standard neoclassical economics competition in an exchange market near equilibrium is mainly driven by bargaining skill, since the quantity and quality of goods available is not changing. In Schumpeter's world, by contrast, competition is driven by competitive advantage resulting from innovation by `first movers', imperfect information, and consequential barriers to entry. Neoclassical economists like Alchian(A. A. Alchian, 1950)and Friedman (Milton Friedman, 1953) argued that Schumpeterian competition is consistent with profit maximization, because only maximizers will be `selected' (in the Darwinian sense) by the market. However evolutionary economists like Nelson and Winter (Richard R. Nelson and Sidney G. Winter, 1974, 1982b, Sidney G. Winter, 1964) point out that the Darwinian `selection' analogy is inappropriate without an inheritance mechanism to assure sustainability of this behavior over time. In any case it is now known that Darwinian selection is very inefficient.

The main difference between evolutionary economics as it has developed so far and the neoclassical mainstream has been characterized as follows: that neoclassical theory postulates `representative' firms operating on the boundary or `frontier' of a well-defined whereas evolutionary biology – and evolutionary economics – lays great stress on the existence of diversity (Jeroen Van den Bergh, 2003), In fact, the mechanism that drives the economic system, in our evolutionary view, is a kind of conflict between diversity and selection. In biology, diversity of populations and species is assured by mutation combined with diversity of environments. In economics it is the result of diverity of talents and ideas among entrepreneurs, together with diversity of environments and external circumstances. The selection mechanism in biology is called `survival of the fittest', although the details of what constitututes `fitness' are still very unclear. In evolutionary economics e.g. (Richard R. Nelson and Sidney G. Winter, 1982a) selection is essentially defined as survival as a viable competitor in the market.

Nelson and Winter have shown that a plausible growth process can be simulated by postulating a population of firms not in equilibrium, displaying bounded rationality, and interacting with each other on the basis of probabilistic rules (Richard R. Nelson and Sidney G. Winter, 1982a). However, it is extremely unlikely that their population-based approach will ever be usable for macro-forecasting, which is the ultimate test of any scientific theory.

Hence, we seek another approach. The standard model, which assumes that energy and other natural resource inputs contribute very little to production (because of their negligible role in the national accounts), contradicts economic intuition. Economic history suggests that increasing natural resource (energy) flows are indeed a major factor of production and that declining costs – in relation to the rising wages of labor – have induced ever-increasing substitution of machines (consuming fossil fuels) for human labor. This long-term substitution has evidently been a key driver of economic growth, especially since the industrial revolution.

Hence our approach treats the economy as an evolutionary materials processing system. The system consists of processing stages, starting with extraction, conversion, production of finished goods and services, final consumption and disposal of wastes. An adequate description of the system must include materials and energy flows as well as money flows . These flows and conversion processes are governed by the laws of thermodynamics. At each stage, until the last, mass flows are split by technological means into `useful' and waste categories. Value (and information) is added to the useful flows, reducing their entropy content and increasing their exergy content per unit mass (thanks to exogenous inputs of exergy), while the high entropy wastes are returned to the environment.

From a very macroscopic perspective the system can be decomposed into a product of successive conversions from primary resource inputs to intermediate and final products and , with corresponding efficiencies, viz.

$$GDP = R \times \frac{IO_1}{R} \times \frac{IO_1}{IO_2} \times \frac{IO_2}{IO_3} \times \cdots \times \frac{GDP}{IO_n} \qquad (1)$$
$$= R \times f_1 \times f_2 \times \cdots g$$

where $\boldsymbol{f_1}$ is the conversion efficiency of the resource (exergy) inflow $\boldsymbol{R}$ into the first level intermediate product, $\boldsymbol{f_2}$ is the conversion efficiency to the second level intermediate product, and so forth. The term $\boldsymbol{g}$ is just the ratio of output to the last intermediate product. This expression is an identity. When we define the levels and intermediate products (later) it will become a model.

The term *energy* as used above, and in most discussions (including the economics literature) is technically incorrect, since energy is a conserved quantity. It therefore cannot be `used up' but only converted from available to unavailable forms. The correct term in this context is *exergy*, which is roughly speaking, `available energy' or `potentially useful energy'.[2] Generally speaking it is accurate to say that exergy is simply the correct word for `energy', as the latter word is used by most laymen in most situations. However exergy is definable for all materials – not only fuels. It is therefore a common measure for both `energy carriers' and other materials.[3]. The exergy of a material is defined in terms of `distance' from a reference state with which to which the material must ultimately reach thermodynamic equilibrium (usually the atmosphere, ocean or earth's crust) after all possible spontaneous (endothermic) reactions – such as combustion or corrosiont – have taken place.

Energy and work as factors of production

Efforts to explain growth in terms of energy consumption have a long history, going back to the 19th century Monist movement, one variant of which was `general energetics' propounded, especially, by the German chemist and Nobel laureate, Wilhelm Ostwald. He argued that cultural changes are driven by changes in energy technologies, and vice versa. Some of his contemporaries (including Ernest Solvay) went further and advocated an energy theory of value, i.e. that value must be proportional to embodied energy. All of this was swept aside by the `subjectivist' revolution in economics, which arose from dissatisfaction with `substance' theories arising from such difficulties as their inability to account for different values for the same goods under different circumstances.

Frederick Soddy, another Nobel chemist, re-visited the question in the 1920s, distinguishing between `real wealth' based on material goods and `virtual wealth' created by financial transactions (Herman E. Daly, 1980, Frederick Soddy, 1922, 1933). Soddy's work was one of the inspirations of the `technocratic movement' in the 1930s, supported by Thorstein Veblen. The technocrats did adopt an energy theory of value, per se, but they did aim at replacing the market economy with a `scientific' rule based on energy certificates, rather than money (J. Knoedler and A. Mayhew, 1999, D.R. Stabile, 1986).

More recently the energy theory of value has been revived once again, by the ecologists H.T. Odum and his students (Robert Costanza, 1982, 1980, Robert Costanza and R. A. Herendeen, 1984, Howard T. Odum, 1977, 1973, Howard T. Odum and E. C. Odum, 1976). Their approach has undergone some refinement and revision, and emerged as the `biophysical approach' (Cutler J. Cleveland, 1992, Cutler J. Cleveland et al., 1984, Cutler J. Cleveland et al., 1998, Cutler J. Cleveland and Matthias Ruth, 1997). For a subjectivist economic critique, see (David A. Huettner, 1982).

The so-called biophysical school, sketched above, is still active, albeit not taken seriously by mainstream economists. The mainstream view is that energy and other natural-resource based products should be regarded as economic intermediates, insofar as they are produced by industrial activity. But this is equally and no less true of capital stock. (In fact, the skills and knowledge embodied in the labor force, too, are products of economic activity as well as inputs). Of course, it can be argued that, while capital and labor stocks can be augmented in the future, current economic output is only dependent on the quantities of these factors that currently exist. But the same statement is also true of energy and physical resource flows. They are limited by past investment, both in supply and capacity for utilization. Neither can be increased instantaneously beyond fixed limits. The point is that, to a naive observer, energy and material resources are as much a factor of production as labor or capital. Moreover, it is entirely plausible that resource consumption (or, as it turns out, power consumption) is a reasonable proxy for technical change, or 'technological progress' in Solow's theory.

The possible contribution of natural resource inputs to growth (or to technical progress), within a production-function approach was not considered seriously by economists until the 1970s (mainly in response to the Club of Rome's "Limits to Growth" report (Donella H Meadows et al., 1972)), and then only as a possible constraint (Partha Dasgupta and G. Heal, 1974, Robert M. Solow, 1974, Joseph Stiglitz, 1974). It follows that, in more recent applications of the standard theory (as articulated primarily by Solow) resource consumption has been treated as a consequence of growth and not as a factor of production. This simplistic assumption is built into virtually all textbooks and most of the large-scale models used for policy guidance by governments. We argue *a priori* that the assumption that energy consumption is only a consequence of growth is false. On the contrary, energy (exergy) consumption is as much a driver of growth as a consequence. (Having said this, however, we emphasize that it is exergy *services,* rather than exergy *inputs*, that are the true factor of production.)

The generic energy (exergy) feedback cycle works as follows: cheaper energy and power (due to discoveries, economies of scale and technical progress in energy conversion) enable goods and services to be produced and delivered at lower cost. Thus, exergy converted into mechanical (or chemical) work is `productive'. Lower cost, in competitive markets, translates into lower prices which encourages demand growth. Since demand for final goods and services necessarily corresponds to the sum of factor payments, most of which flow back to labor as wages and salaries, it follows that wages of labor and returns to capital tend to increase as output rises.[4] This, in turn, stimulates the further substitution of mechanical power for human (and animal) labor, resulting in further increases in scale and still lower costs, *ad infinitum*.

Based on both qualitative and quantitative evidence, the positive feedback relationships sketched above imply that physical resources have been, and still remain, a major factor of production and driver of growth. It is not surprising, therefore, that including a resource flow proxy in the neoclassical production function, without any exogenous time-dependent term, seems to account for economic growth quite accurately for significant time periods (Bruce M. Hannon and John Joyce, 1981, Reiner Kümmel, 1989, Reiner Kümmel and Dietmar Lindenberger, 1998, Reiner Kümmel et al., 1985). However, models based on commercial energy (fuel) inputs cannot explain economic growth over a longer time-frame.

Several models have utilized electricity as a `form' of energy. This device achieves better results (Bernard C. Beaudreau, 1995, Cutler J. Cleveland, Robert Costanza, C. A. S. Hall and Robert K. Kaufmann, 1984, Cutler J. Cleveland et al., 2000). Unfortunately, the authors confuse electricity as *work* from electricity as *energy*, thus missing the critical distinction.

Electricity happens to be a product that is manufactured and traded. Most of it is sold commercially, so it coincides with a *sector.* The product has costs and a price. The inputs are heat energy from fuel (fossil or nuclear), as well as labor and capital. Electricity, the product, is essentially pure *work*, in the physical (thermodynamic) sense. Other types of physical work are also produced and consumed within the economy, although they do not coincide with specific sectors. Examples include human and animal muscular work (formerly important on farms), mechanical work produced by mobile internal combustion engines (mostly in transportation equipment and construction or farm machinery) or useful heat as delivered to a point of use.

With regard to capital and labor, it is generally understood that it is not the size of the labor force or the magnitude of the capital stock, but the service flows they provide that matters. (The convention that labor services are proportional to the one, and capital services to the other, has led to a regrettable tendency to confuse the stocks and flows.) In similar vein, we argue that it is not energy (exergy) flows *per se* but the sum of all *exergy services* – or useful workflows, as noted above – that we regard as the important factor of production.

Growth as a physical process

Hereafter, we assume that both resource inputs (***R***) and waste outputs (***W***) from the economy are defined and measured in terms of exergy flows. Using the first law of thermodynamics (mass/energy balance) it is now possible to introduce a measure of the technical efficiency of the production process, namely either the ratio of resource (i.e. exergy) inputs to the output of useful work (in the physical sense) done by the economic system. Energy services are analogous to capital services, as distinguished from energy stocks or the energy content of raw materials such as fossil fuels. The difference between exergy input and useful work (exergy service) output is lost as exergy process waste ***W***. The latter consists largely of waste heat, but it has a material pollution component that is generally harmful to the environment.

For simplicity, we now consider the case where the economy is a two stage sequence, with one intermediate product, to be defined. The technical efficiency of production ***f*** can be characterized as follows:

$$f = \frac{R - W}{R} = \frac{U}{R} \qquad (2)$$

where $U = R - W$ is the intermediate product.

Note that ***f*** is a fraction (i.e. a dimensionless number), limited to the range of values $0 < f < 1$ (since $0 < W < R$). In other words, ***f*** can be interpreted as the technical efficiency of the production process, which is the process of converting resource inputs to useful outputs. As noted, both inputs and outputs can be measured in the same (exergy) units.

It is now convenient to define another new measure, ***g***, as follows:

$$g = \frac{Y}{R - W} = \frac{Y}{U} \qquad (3)$$

The variable ***g*** is the monetary value of the output of the economy (GDP) per unit of useful exergy or exergy services produced. (It can be regarded as a crude measure of the extent of `decoupling', or `materialization' – or dematerialization – of the economy.) Combining (2) and (3) we get

$$Y = Ug = fgR \qquad (4)$$

which is still an identity. However, the right-hand side of (4) can be interpreted as an aggregate production function provided ***g*** is a homogeneous function of labor , capital and exergy services. of order zero. We postulate that ***U*** can be interpreted as `useful work', or `exergy services'.

To anticipate a possible objection, we do *not* assume that firms must operate on, or move along the `frontier' (by substitution among factors) as they would have to do if they were profit maximizers with perfect information in a perfectly competitive market. On the contrary, we postulate (in the spirit of Friedman (Milton Friedman, 1953)) that if an assumed relationship explains (i.e. reproduces) the empirical observations, we need not worry too much about the realism of underlying assumptions.

In short, we argue that a functional relationship among aggregates is an adequate representation of reality for purposes of explaining growth, even though the `frontier' is a fictitious surface in a turbulent cloud of points, not a sharp boundary. Almost all firms are operating at some distance from this hypothetical frontier, some inside and some outside it. The only further assumption needed to account for this picture is that firms do not have perfect knowledge or foresight, and that competition is not perfectly efficient. A firm too far inside the cloudy frontier is likely to be unprofitable and risks being selected out in due time. Such a firm may be forced to change its products or its operating systems, in order to survive – in short to innovate. On the other hand a firm on the outside of the fuzzy frontier is likely to be profitable and to grow at the expense of others, at least in the short term.

The initial assumption of a single sector with a single all-purpose product is oversimplified of course, because it allows no scope for innovation. In our picture the product of the economy is an extremely heterogeneous collection, such that it would be exceptional for two firms to produce exactly the same product or service by the same technique.

The right-hand side of (4) can be interpreted as an aggregate production function, supposing ***Y = Y(L,K,R)***. In this case ***g(L,K,R)***, whence ***f(t)*** can be interpreted as the so-called "Solow residual", namely the multiplier that represents technological progress. Alternatively, however, we can suppose that ***Y = Y(L,K,U)***, treating ***U*** as a factor of production, rather than an intermediate. (***U*** can be interpreted as exergy services, just as labor ***L*** is normally interpreted as labor services, rather than labor stock, and capital ***K*** is similarly interpreted as capital services, not capital stock.) In this case, (4) can be regarded as a production function provided only that ***g*** is a homogeneous function of order zero. This condition is satisfied, for instance, provided ***g*** depends only on ratios of other factors of production.

The generic form of the production function is $Y = f(K,L,U)$, where

Y = output (GDP, $)

K= industry capital (value of stocks, $)

L = labor (hours of work, hrs)

U = exergy services (work inputs, EJ)

It is convenient to index these variables to their values in the starting year, 1900, viz.

$$\begin{aligned} y &= Y/Y_0, \\ k &= K/K_0, \\ l &= L/L_0, \\ u &= U/U_0 \end{aligned} \qquad (5)$$

The total time derivative of the normalized production function **y** can then be expressed as

$$\frac{dy}{dt} = \left(\frac{\partial y}{\partial k}\right)\frac{dk}{dt} + \left(\frac{\partial y}{\partial l}\right)\frac{dl}{dt} + \left(\frac{\partial y}{\partial u}\right)\frac{du}{dt} + \frac{\partial y}{\partial t} \qquad (6)$$

However, our objective is to explain production and growth as endogenous processes, hence without any time dependent. Henceforward we assume the ***y*** has no explicit time dependence. Dividing through by ***y*** the equation (1) can be rewritten as follows:

$$\frac{d\ln y}{dt} = \alpha\frac{dk}{dt} + \beta\frac{dl}{dt} + \gamma\frac{du}{dt} \qquad (7)$$

where the terms α, β and γ are output elasticities (marginal productivities) of capital ***k***, labor ***l*** and exergy services (work) ***u*** respectively. The three factor productivities are logarithmic derivatives of the arguments, viz.

$$\alpha = \left(\frac{k}{y}\right)\left(\frac{\partial y}{\partial k}\right) = \frac{d\ln y}{d\ln k}$$

$$\beta = \left(\frac{l}{y}\right)\left(\frac{\partial y}{\partial u}\right) = \frac{d\ln y}{d\ln l} \qquad (8)$$

$$\gamma = \left(\frac{u}{y}\right)\left(\frac{\partial y}{\partial u}\right) = \frac{d\ln y}{d\ln u}$$

We impose the usual condition of constant returns to scale,

There are two possible ways to determine a plausible mathematical form for a production function. One is to choose the simplest possible multi-parameter mathematical form that satisfies the constant returns and integrability conditions above, plus any other condition that seems desirable, and perform a statistical fitting operation to select optimum values of the parameters. This approach has been used in the past for two-factor production functions involving capital and labor only. The Cobb-Douglas production is, essentially, the simplest mathematical form that satisfies the Euler (constant returns to scale) and integrability conditions[5], while the CES function also satisfies a further condition, namely constant elasticity of substitution between the two factors. Both functions assume constant elasticities of output (factor productivities) over time. This assumption is plausible for the two factor case, for moderate time periods, but not for a three (or four) factor case and a very long period of time.

The second approach, which we favor, is to start with flexible mathematical forms for the factor productivities, but imposing appropriate asymptotic conditions. One can subsequently perform a partial integration to obtain the corresponding aggregate production function. The last step, as before, is a statistical fit to determine the best values of the parameters. This approach seems more appropriate given the long period of time over which we hope to explain economic growth.

The following functional forms for factor productivities were first proposed by Kümmel et al (Reiner Kümmel, Wolfgang Strassl, Alfred Gossner and Wolfgang Eichhorn, 1985), with one critical difference. In our version, we substitute useful work (exergy services) ***u*** for exergy inputs ***e***. The proposed marginal productivities are as follows:

$$\begin{aligned} \alpha &= a\left(\frac{l+u}{k}\right) \\ \beta &= a\left[b\left(\frac{l}{u}\right) - \left(\frac{l}{k}\right)\right] \\ \gamma &= 1 - \alpha - \beta \end{aligned} \tag{9}$$

Partial integration yields the following linear-exponential (LINEX) function:

$$y = u\exp\left[a\left(2 - \left(\frac{l+u}{k}\right)\right) + ab\left(\frac{l}{u} - 1\right)\right] \tag{10}$$

Comparing (10) with (4), it is clear that

$$g = \exp\left[a\left(2 - \left(\frac{l+u}{k}\right)\right) + ab\left(\frac{l}{u} - 1\right)\right] \tag{11}$$

According to (2) the variable ***U*** is simply the product of the resource flow ***R*** times the exergy conversion efficiency ***f***. Both can be estimated independently from published technical and historical data (although the estimation procedure is complicated and somewhat imprecise (Robert U. Ayres and Benjamin Warr, 2003). For our purposes hereafter, it is sufficient to know that ***f*** is a monotonically increasing function (except for a very brief excursion during the early 1930s) that takes values between zero and unity.

During recent decades information, computer and telecommunications (ICT) technologies have increased in importance. To reflect this shift, we have introduced a fourth factor of production, capital invested in information technology, denoted $\boldsymbol{k_{ict}}$. It seems plausible that information technology effectively increases the productivity of labor, while correspondingly reducing the productivity of exergy services. For convenience we now define a new term ***cR***, where ***R*** is defined as the ratio of ICT capital stock to total capital stock:

$$cR = c\left(\frac{k_{IT}}{k}\right) \tag{12}$$

The value of ***R*** for the period 1960 to 2000 was calculated from data provided by Jorgenson and Stiroh (Dale W. Jorgenson and Kevin J. Stiroh, 2000), and represents perhaps the most complete and up-to-date estimate of its type. Inserting the modified marginal productivities 16into equation (7) and integrating, we get the same production function as before, namely (10), except for a multiplier, viz. $\left(\frac{I}{u}\right)^{cR}$. This production function will be referred to as the ICT adjusted LINEX.

Empirical results

We have tested the LINEX (10) and its ICT adjusted form to predict empirical GDP ***y,*** for the US, using historical time series for the usual factors, capital and labor, and physical work (*figure 1*). The capital and labor series are taken from standard sources (US Department of Commerce, Business Statistics). The ratio of information capital, and information capital services to total capital stock (services) required for the ICT adjustment (12) were taken from recent work by Jorgenson and Stiroh (Dale W. Jorgenson and K. J. Stiroh, 1995) and are plotted in *Figure 2*. The exergy series for the US was developed in prior published work by the authors (Robert U. Ayres and Benjamin Warr, 2003). Energy (exergy) conversion efficiencies were also taken from this study and used to estimate the physical work obtained from all exergy sources. *Figure 3* shows the allocation of exergy in the US economy among major uses (muscle work, high temperature heat, low temperature heat, propulsion and electric power generation) from 1900 through 1998. *Figure 4* shows our estimates of the average conversion efficiency, over time, based on the history of technology. Combining the two calculations yields the physical work estimate (*figure 1)* and the overall efficiency trend *Figure 5*, here plotted with the work/GDP ratio *(1/g)*. A simple way of modeling the f-function parametrically is shown in Appendix A. This may be helpful in cases where detailed historical data are unavailable.

The fitting was done by constrained non-linear optimization (minimization of mean-square error) for two periods, viz. 1900 through 1980 and 1900 through 1998, both with and without the adjustment for information technology. However, it is worth noting that the ICT adjustment is quantitatively insignificant prior to the early 1980s. *Table 1* shows the fitted parameter values and quality of fit statistics and *figures 6-9* the fitted estimates. Issues when fitting the data are discussed in Appendix B.

The calculated production function with three factors (labor, capital and exergy services or physical work) and two parameters ***a, b,*** fit over either time period explains economic growth very well until 1970. However there is a fairly sharp change starting shortly after that time; empirical GDP is slightly overestimated during the period 1970-80 and underestimated thereafter. Structural changes are the likely cause. The first indication of structural change is the observation that the ratio of work/GDP which had been rising more or less monotonically throughout the century, peaked in 1970-72 and reversed direction thereafter rather sharply, as shown in *Figure 5*. Given the well-known "energy crisis" of 1973-74 it seems likely that the sharp rise in energy prices that began then was primarily responsible for the reversal phenomenon. The second indication of change is that real growth lagged potential (i.e. predicted) growth significantly in the 1970s and early 80s. The most plausible explanation of this is that the sudden energy price increases resulted in an immediate decline in demand below the previous trend line, and consequent (short-term) over-supply of certain types of energy-related capital goods such as electric generating facilities, aluminum reduction facilities and the like. When the capital stock adjustment ran its course, the predicted GDP and the actual GDP followed more nearly parallel tracks, although the penetration of ICT was increasingly making itself felt.

The optimal values of the parameters ***a, b*** were ***a = 0.13*** and ***b = 3.40***. In the first case, when the optimization was done from 1900 through 1980 using (10) and extrapolated without adjusting for ICT effects, the root mean-square error was 0.404 for the entire period (1900-1998) and 0.374 for the shorter 1900-80 period. Evidently, it is possible to explain US economic growth since 1900 with surprising accuracy without explicitly adjusting for ICT effects. However the adjustment for ICT improved the fit obtained after 1980. The parameters ***a, b*** were unchanged, but the additional parameter ***c*** was chosen to force the new production function to match the actual data for 1990. The resulting value for ***c*** was -0.0006. Using the adjusted 3-parameter production function, the mean square error for the period 1900-1980 was 0.399, and for the entire period, including the extrapolation to 1998, it was 0.380. Both predictions of GDP for the period 1900-1998 are presented in *figure 6.* The results are shown in greater detail in *figures 7-9*, for 3 arbitrary 35 year periods. The final plot shows how the inclusion of ICT effects has improved the estimate after since 1980 when the productivity effects of ICT can no longer be assumed to negligible.

Concluding comments

The first reaction of a skeptical reader is likely to be that the results shown here are too good to be true. Therefore it would seem that we must have cheated, by somehow assuming the answer. We did not. Our consciences are clear on this point. The conversion efficiency calculations on which the rest depends were done (and published) first. While the calculations are undoubtedly flawed because the historical data are incomplete and imperfect, they were done as carefully as we know how.

However, the persistent skeptic could argue that, after all, our efficiency trend is a direct consequence of technological progress, hence is really not a conceptual improvement over the Solow residual. Why, she may well ask, is our technical conversion efficiency trend any more `endogenous' than Solow's multiplier? The answer, as before, is that we calculated the efficiency trend directly from historical data on energy (exergy) consumption and conversion efficiencies, not indirectly by working back from GDP and an assumed production function.

A further remark is appropriate here. A pactical difficulty is that many of the efficiency numbers needed for the theory are not compiled or published by any government agency. This, in turn, is because useful work (in the physical sense), in general, is not a well-defined commodity that is produced by a defined sector and sold to other sectors. Electric power produced by central stations (utilities) is the single exception to this rule. In this case both inputs and outputs are recorded and published annually by the Federal Power Commission and the Department of Energy. The conversion efficiency in this case is easily calculated. It is unfortunate that comparable data for other (non-electric) forms of useful work are not routinely collected and published at present.

The next skeptical comment could be that our line of argument does not constitute a truly endogenous theory of growth because we have not postulated an explicit economic mechanism to explain the technological progress implicit in the efficiency trends. Here the skeptic is on slightly firmer ground.

It is true, for instance, that we cannot explain in purely economic terms why the efficiency of electric power generation has increased so much more rapidly than the efficiency of space heating or automotive engines. Actually, several hypotheses do come to mind, but we admit that we have not seriously tried to explain this phenomenon. (We remind the reader that technological progress in the real world does not occur uniformly and smoothly across all sectors, as it would if technological progress were truly exogenous.) We could even remark that the spillovers from increased efficiency and reduced costs of electric power generation have undoubtedly been far greater than any imaginable spillovers from more efficient insulation. But such comments are really beside the point. The essential point is that we calculated the efficiency trend first, from real technical (and historical) data, whereas the Solow residual is simply unexplained.

The next and most troublesome criticism is probably the following: Are you really saying that the ONLY technologies that contribute to economic growth are technologies that increse the efficiency of exergy conversion? You are suggesting, for instance, that medical technologies do not contribute to economic growth, even though such spectacular progress has been made in conquering disease. .

The answer to that is a firm yes and no. Yes, we do, in fact, suggest that health services – along with other labor-intensive services – add little to measurable growth. They suffer from what has been called the Baumol disease. Inputs to health services, as well as education, government, finance and other services, are mostly labor, and it is difficult to measure outputs independently, except in terms of input costs.

No, we are not suggesting that the only growth enhancing technologies are innovations that increase the efficiency of primary conversion. In fact, the record since 1975 or so suggests otherwise. As equation (1) indicates, we view the economy as a set of linked materials-processing and value added stages, forming a sequence. Equations (2-12) refer to a simplified situation, with only one intermediate product, namely useful work, ***U***. The surprise is that this simplification proves to be such a good approximation to the complex reality.

However, the approximation shows signs of breaking down after 1975, and the underlying reason is not hard to guess. Until 1972, (and also since 1987 or so) resource prices were on a long-term down escalator, while labor prices were on a corresponding up escalator. For this reason, resource-intensive manufacturing businesses invested their capital with the specific objective of reducing labor requirements. They did this by investing heavily in exergy-intensive machines and equipment, and of course the feedback cycle (*Figure 1*) operated to keep the costs of primary exergy services (useful work) declining.

From 1973-1986 the feedback cycle went partially into reverse, and the economy actually shrank for a few years. During this period, for the first time, resource prices rose sharply and there were significant efforts to reduce consumption of resources *per se*. Since the opportunities for short-term increases in primary exergy conversion efficiency were quite limited, by that time, attention switched to finding cheap ways to decrease the consumption of electric power, and automotive fuels. Opportunities to save energy at low cost, or even at a profit, by improved `housekeeping' were widely available. Old and new buildings were better insulated, double and triple windows were installed, incandescent lights were replaced by compact fluorescent lights in many homes, refrigerators and air-conditioners were significantly improved – partly by the use of better insulation – and the so-called CAFE standards adopted by the US Congress, forced auto companies to double fleet average fuel economy over ten years (mainly by cutting the size and weight of new vehicles). These savings can be interpreted as increasing *secondary efficiency*, in the sense of equation (1).

It can be expected that future gains in productivity will increasingly be achieved by improving secondary and (in due course) tertiary efficiency. Tertiary efficiency can be thought of as the efficiency with which secondary work produces finished goods and services.

In conclusion, we would like to emphasize a crucial point once again. The neo-classical paradigm does not allow for `real' material flows. Production and consumption are abstractions, linked only by money flows, payments for labor, payments for products and services, savings and investment. Resources are also abstractions. These abstract flows are governed only by equilibrium-seeking market forces (the "invisible hand"). The laws of physics play no role. According to this paradigm, there is no deep fundamental connection between the physical world and the economy. Technological progress is exogenous, like `manna from heaven'.

Indeed, in the neoclassical paradigm long-term economic growth is simply assumed. It follows from this assumption that our grandchildren will be a lot richer than we are, whatever we do or do not do about global environmental problems such as climate warming (or `climate chaos'). Under these (supposed) circumstances, the obvious policy is to continue business-as-usual, in the belief that it is optimal to go on doing the things that made us rich in the first place. If it ain't broke, don't fix it!

On the other hand, the evolutionary paradigm treats the economy as a materials processing system, albeit governed by the laws of supply and demand as well as the laws of thermodynamics. The system consists of processing stages, starting with extraction, conversion, production of finished goods and services, final consumption and disposal of wastes. A description of the system includes materials and energy flows and gradients as well as money flows and price gradients. In this paradigm waste flows are inherent to the economic system. Moreover, the damages from waste flows to and buildup in the environment can both reduce human welfare directly and increase the cost of finding resources, extracting, processing and disposing of waste materials.

In this paradigm future human welfare, and perhaps even survival, depends upon adopting proactive policies and strategies to seek and, if necessary, subsidize more benign alternatives to the business-as-usual path on which we find ourselves.

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Appendix A: A simplified approach to efficiency estimation for other countries.

The calculation of ***f*** from historical data is difficult or impossible for many countries. However, we can estimate a plausible general form for the function and this opens the possibility of estimating ***f*** on the basis of similarities and differences from US experience, requiring much less detailed information.

The rising trend of technical efficiency ***f*** is certainly an elongated S-curve, but not necessarily a symmetric one. In other words, the point of inflection where the second derivative vanishes need not occur at the halfway point, (as it does in the special case of the logistic curve). A suitable functional form for ***f*** that is readily estimated can be derived from the Mahajan-Schoeman differential equation for generalized diffusion processes(Vijay Mahajan and M. E. F. Schoeman, 1977, Christos Skiadas, 1985):

$$\dot{f} = a(1 - f) + b(1 - f)f \qquad (13)$$

which has the solution

$$\ln\left(\frac{1 - f}{a + bf}\right) = -(a + b)(c + t) \qquad (14)$$

and the inflection point

$$f_{in} = \frac{b - a}{2b} \qquad (15)$$

which can be solved for ***a*** or ***b*** when f_{in} is known (or vice versa). Or the two parameters can be determined by some other means, such as a least squares fit. However, an approximate fit can easily be obtained by introducing values of ***f*** for (say) t = 1900 and t = 1998.

Appendix B: Statistical measures of the quality of fit

Concerns are often raised about collinearity when fitting functions whose variables are all explicit functions of time. Collinearity describes the existence of strong linear relationships between the explanatory variables and is common in economic data. The basic problem is that when two (or more) of the explanatory variables move together it is not possible to determine their separate influences on the variable of interest. Ordinary least squares is the most widely used technique for the identification of production function parameters. The Cobb-Douglas is the most commonly used 2-factor production function taking raw factors as inputs and fit using ordinary least squares. When using ordinary least squares collinearity may mean that the regression coefficients cannot be consistently estimated, because there are many parameter values that satisfy the function. Placing constraints, for example constant returns to scale, on the permissible solutions to the function can help reduce such negative effects.

Collinearity can be detected by looking at the correlations between variables. The correlation matrix of the input variables (*table B1*) illustrates the strength of the correlation between the traditional factors of production indicating the possibility of the presence of collinearity, such that problems may arise during the fitting process. A common way to overcome this problem is to work with the increments of the raw variables, to remove any first-order trend in the data. The figures in brackets (*table B1*) show the correlation coefficients between the detrended variables. Where the value was unchanged by detrending, no new value is shown. Clearly detrending has reduced the correlation between GDP and the factors (column 1), but not between the factors themselves, or ratios of the factors that are the actual inputs to the LINEX function.

Collinearity is imprecisely defined and as such it is difficult, simply by looking at these statistics, to determine whether problems with the fit will occur or not. A heuristic evaluation of the possible effects is required. The LINEX function itself does not take only raw factors as inputs but ratios of the factors (*figure B1*). The correlation coefficients between these variables are much lower (*table B1*), and detrending does not alter these values. Therefore in the absence of any indications of problems when fitting, these results were considered adequate to confirm that collinearity did not render the parameter estimates invalid. This feature has to be considered a major advantage of the LINEX over other forms of the production function that take raw factor inputs as explanatory variables.

Quality of fit statistics, the root mean square error and Durbin-Watson statistic were used to compare fits. The root mean square error provided a measure of the overall fit,

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} \left(e^2\right)}{n-k}} \qquad \text{(A1)}$$

It is a measure of the absolute deviation of the theoretical fit from the empirical curve expressed in the original units of measurement, where n is the number of samples, k the number of parameters and e the residual from the fitted curve. The Durbin-Watson statistic was used to check for the presence of correlated residual error. It is calculated as,

$$DW = \frac{\sum_{t=2}^{n} \left(e_t - e_{t-1}\right)^2}{\sum_{t-1}^{n} e^2} \qquad \text{(A2)}$$

Again *e* is the residual error, calculated for each year *t,* for a time period of length *n,* where *k* is the number of independent variables. The *DW* statistic takes values between 0 and 4. An optimal value of 2 indicates that there is no significant correlation between the residual error values for each year. The *DW* deviates from this value if the predictions systematically over or underestimate.

Notes

1. N.B. the national accounts reflect payments only to capital (as interest, dividends, rents and royalties) and to labor (as wages and salaries). The accounts therefore do not explicitly reflect payments to inputs (e.g. energy, raw materials or environmental services from `nature'). It is possible, of course, to distinguish payments to some tangible resource owners (royalties), and to natural resource extraction labor, but these payments constitute only a very small percentage of the total.

2.The proper definition of exergy is the maximum work that can be done by a system as it approaches thermodynamic equilibrium with its surroundings, reversibly. Thus exergy is effectively equivalent to *potential work*. There is an important distinction between *potential work* and *actual work done* by animals or machines. The conversion efficiency between exergy as an input and actual work done, as an output, is vital to this paper.

3.This approach to resource accounting has been proposed, in particular, by Wall (1990). By the same token, the aggregate output of useful products, as well as the generation of material wastes, can also be expressed, separately, in exergy terms (Ayres *et al.* 1998).

4.Marx believed (with some justification at the time he wrote) that the gains would flow mainly to owners of capital rather than to workers. Political developments have changed the balance of power since Marx's time. The division between labor share and capital share has been remarkably constant over many decades, although the capital share has been increasing in recent years. However, whether the gains are captured by labor or capital does not matter: in either case, returns to energy (or natural resources) tends to decline as output grows. This can be interpreted as a declining real price.

5. Integrability conditions imply coupled partial differential equations for α, β and

$$k\frac{\partial\alpha}{\partial k} + l\frac{\partial\alpha}{\partial l} + u\frac{\partial\alpha}{\partial u} = 0$$
$$k\frac{\partial\beta}{\partial k} + l\frac{\partial\beta}{\partial l} + u\frac{\partial\beta}{\partial u} = 0 \qquad (16)$$
$$k\frac{\partial\gamma}{\partial k} + l\frac{\partial\gamma}{\partial l} + u\frac{\partial\gamma}{\partial u} = 0$$

and coupling equations that constrain the final form of the production function

$$k\frac{\partial\alpha}{\partial k} = u\frac{\partial\alpha}{\partial u},$$
$$k\frac{\partial\beta}{\partial k} = l\frac{\partial\alpha}{\partial l} \qquad (17)$$

	1900-2000	*1900-1980*	*Service Adjusted*
rmse	0.404 (0.374*)	0.421 (0.334*)	0.380 (0.399*)
DW	0.16	0.16	0.18
R^2	0.994	0.995	0.996
a	0.12	0.13	0.13
b	3.5	3.4	3.4
c			-0.0006

Table 1. Optimal parameter values and quality of fit statistics (see Appendix B).
* Figures in brackets calculated for the period 1900-1980

	y		k	l	r	u_r	l/u_r	$l+u_r/k$
y	1.00							
k	1.00	(0.52)*	1.00					
l	0.99	(0.53)	0.98	1.00				
r	0.98	(0.53)	0.97	0.97	1.00			
u_r	0.99	(0.54)	0.99	0.97	1.00	1.00		
l/u_r	-0.75	(-0.38)	-0.73	-0.78	-0.81	-0.77	1.00	
$l+u_r/k$	0.65	(0.43)	0.59	0.66	0.75	0.70	-0.85	1.00

Table B1. Correlation coefficients between factors of production and inputs to the LINEX function. *Figures in brackets are for increments of the variables if different from those of raw variables.

Figure 1. Factors of production, USA 1900-1998

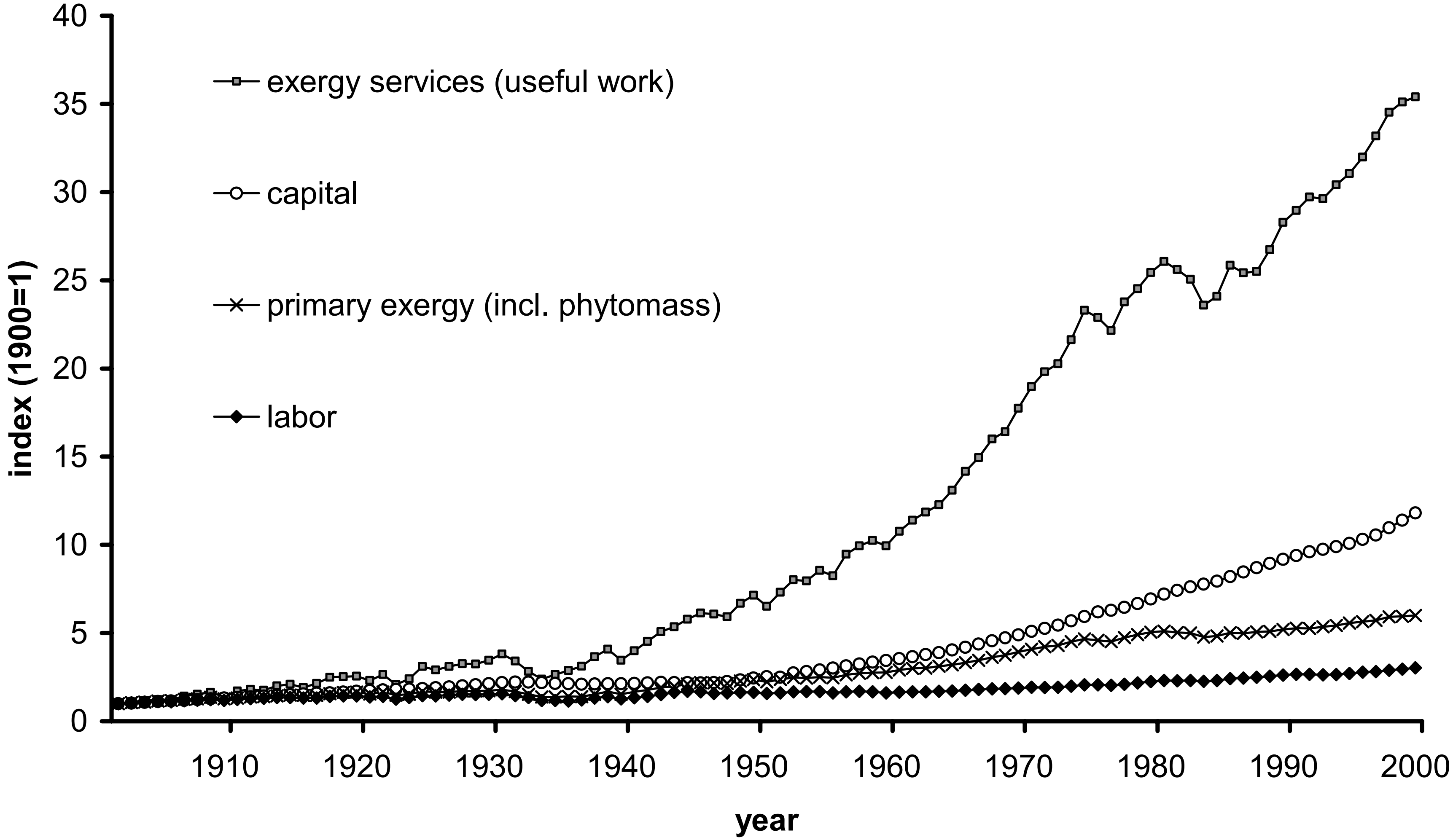
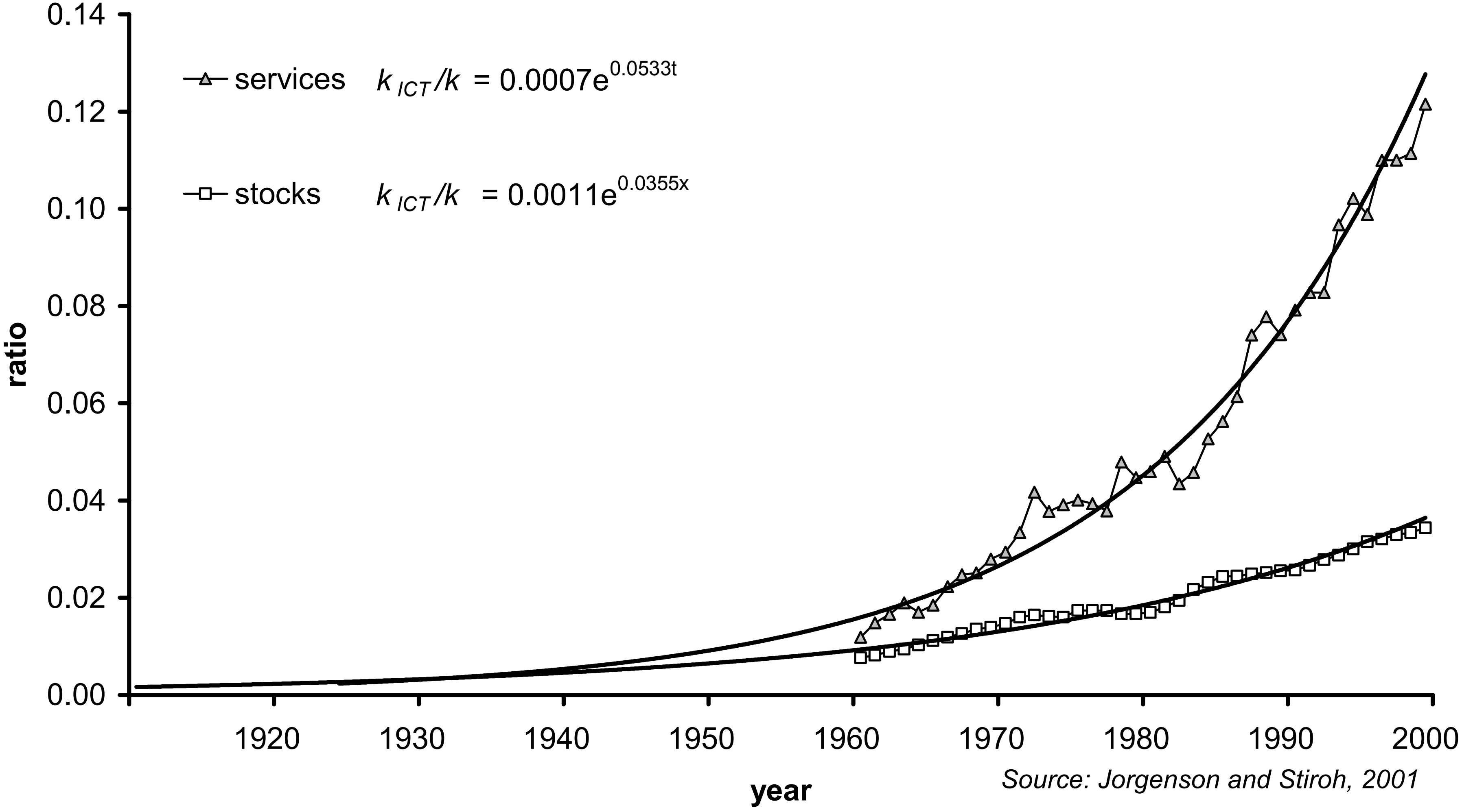


Figure 2. Ratio of value of ICT to total capital, US 1960-2000



Source: Jorgenson and Stiroh, 2001

Figure 3. Primary exergy allocated to types of exergy service (useful work), USA 1900-2000

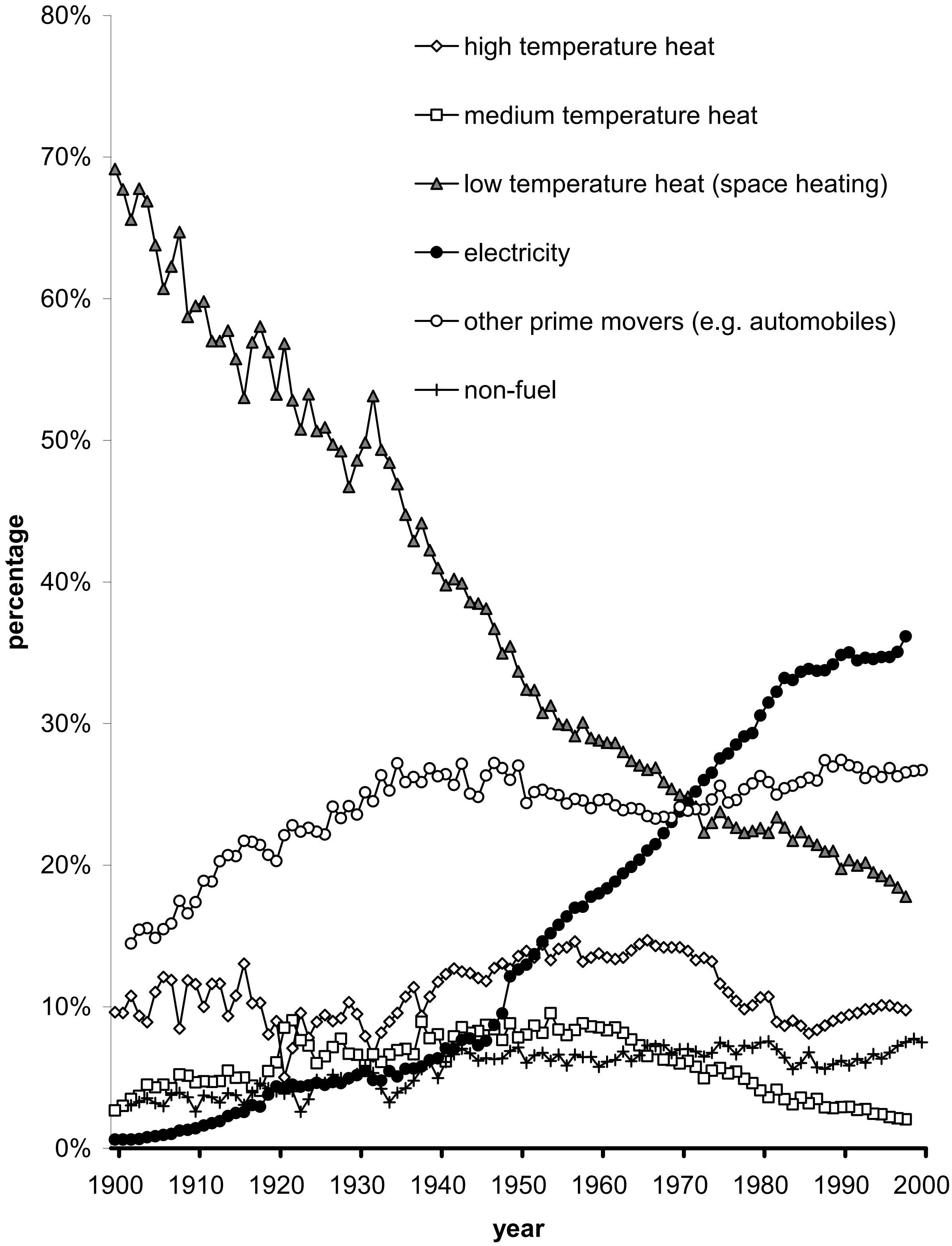


Figure 4. Primary exergy to exergy service (useful work) conversion efficiencies, USA 1900-1998.

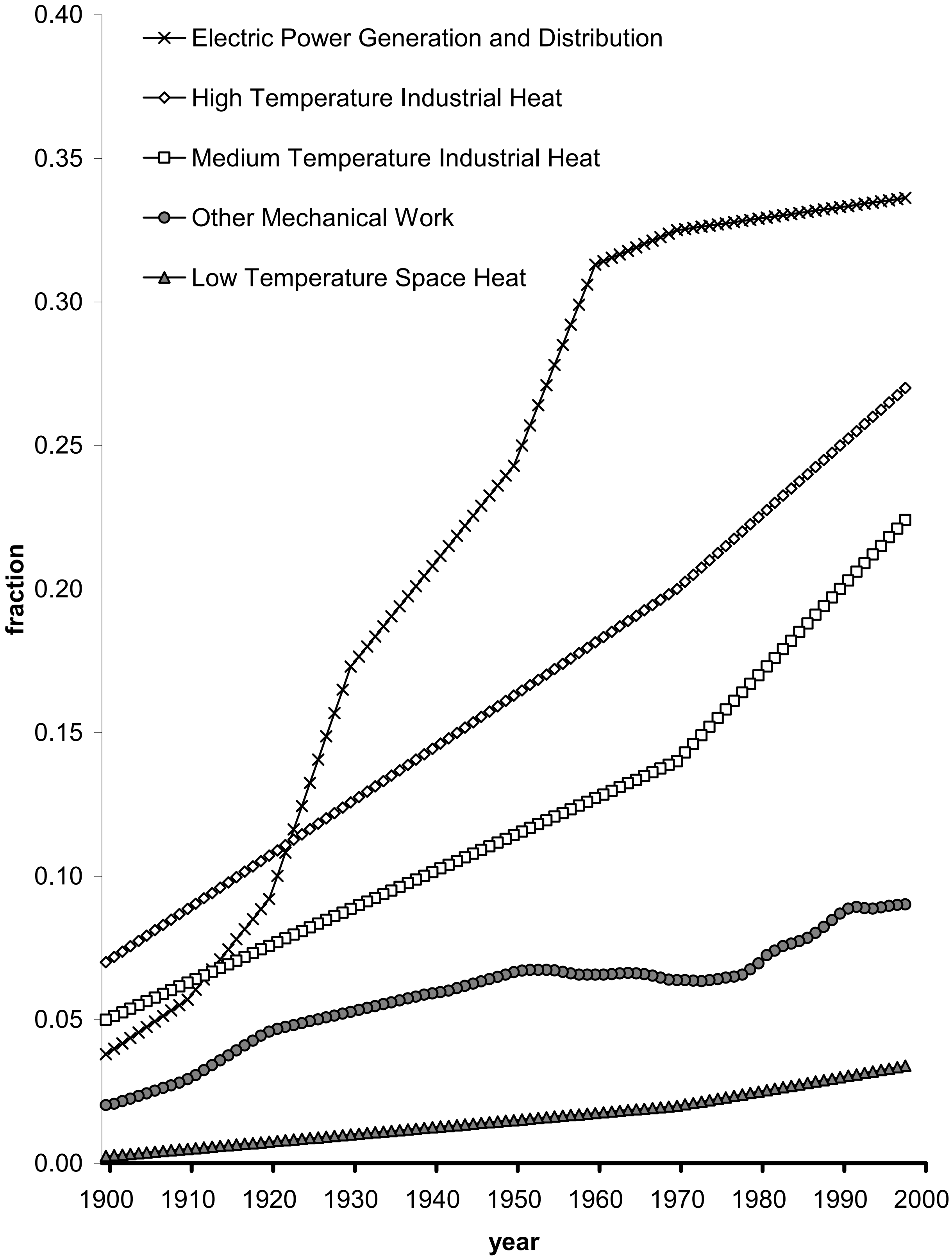


Figure 5. Overall exergy conversion efficiency *f*, and work per unit GDP (*1/g*), USA 1900-1998.

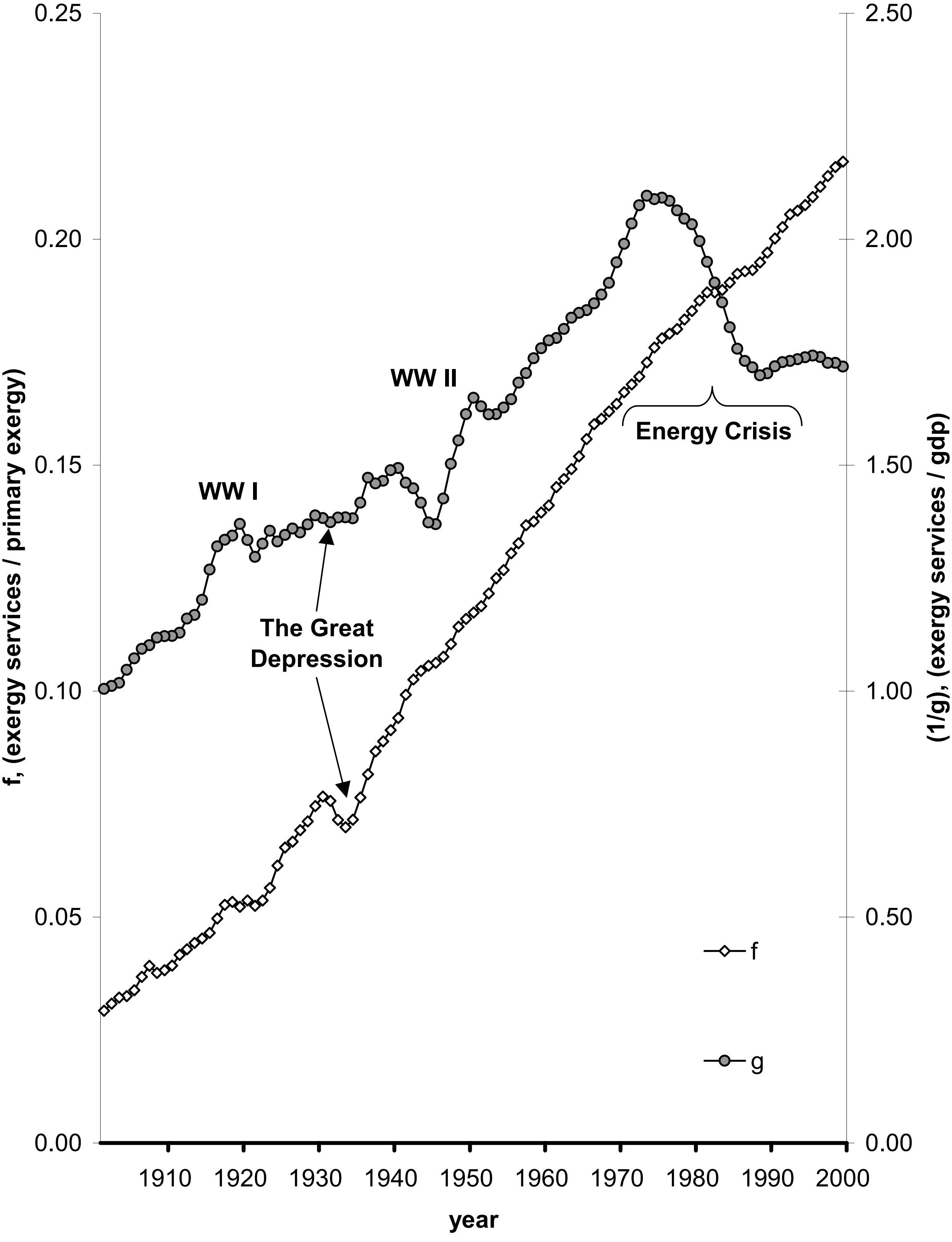


Figure 6. LINEX estimates of output both unadjusted and adjusted for ICT effect, USA 1900-2000.

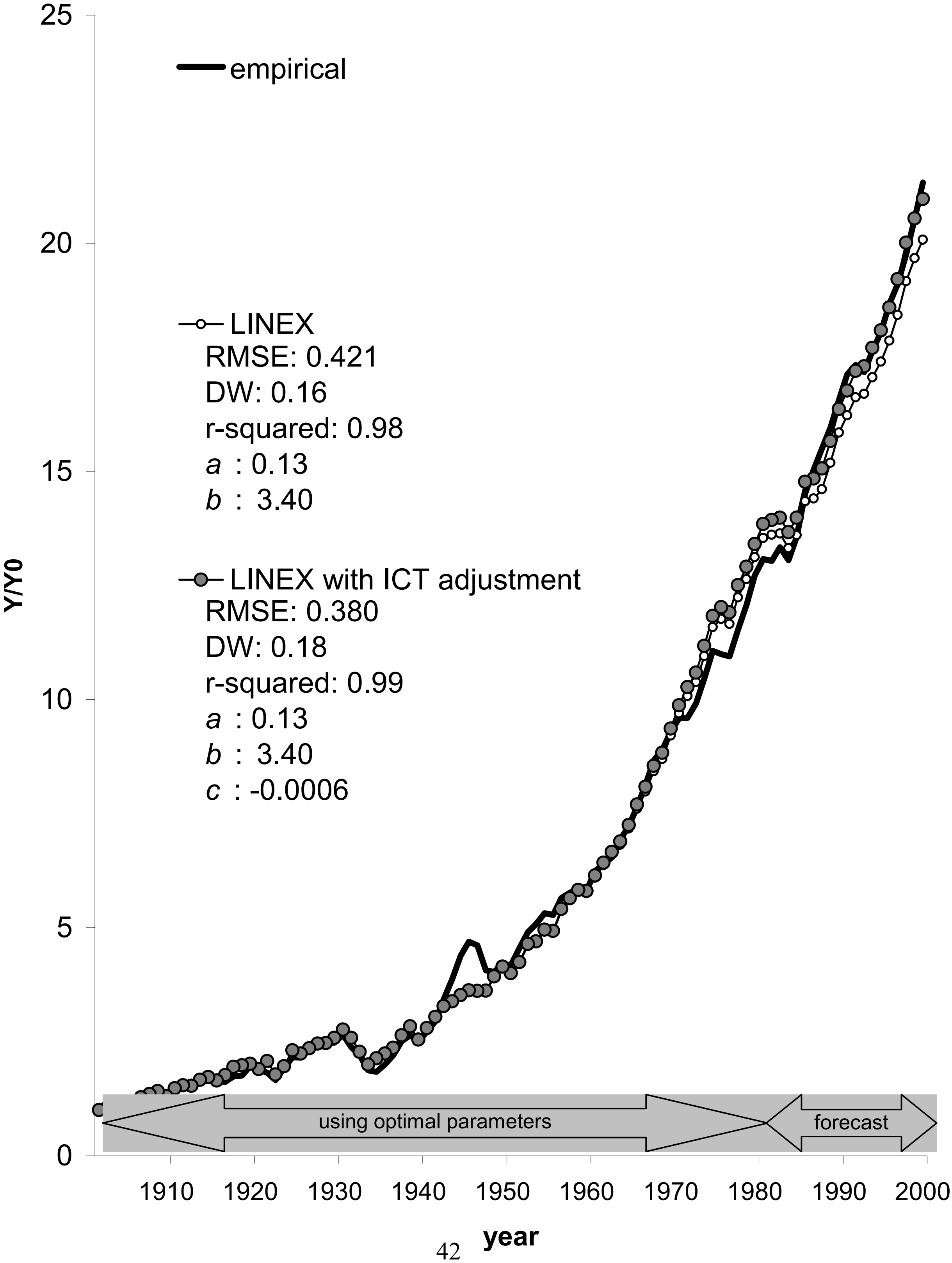


Figure 7. LINEX estimates of output, USA 1900-1935

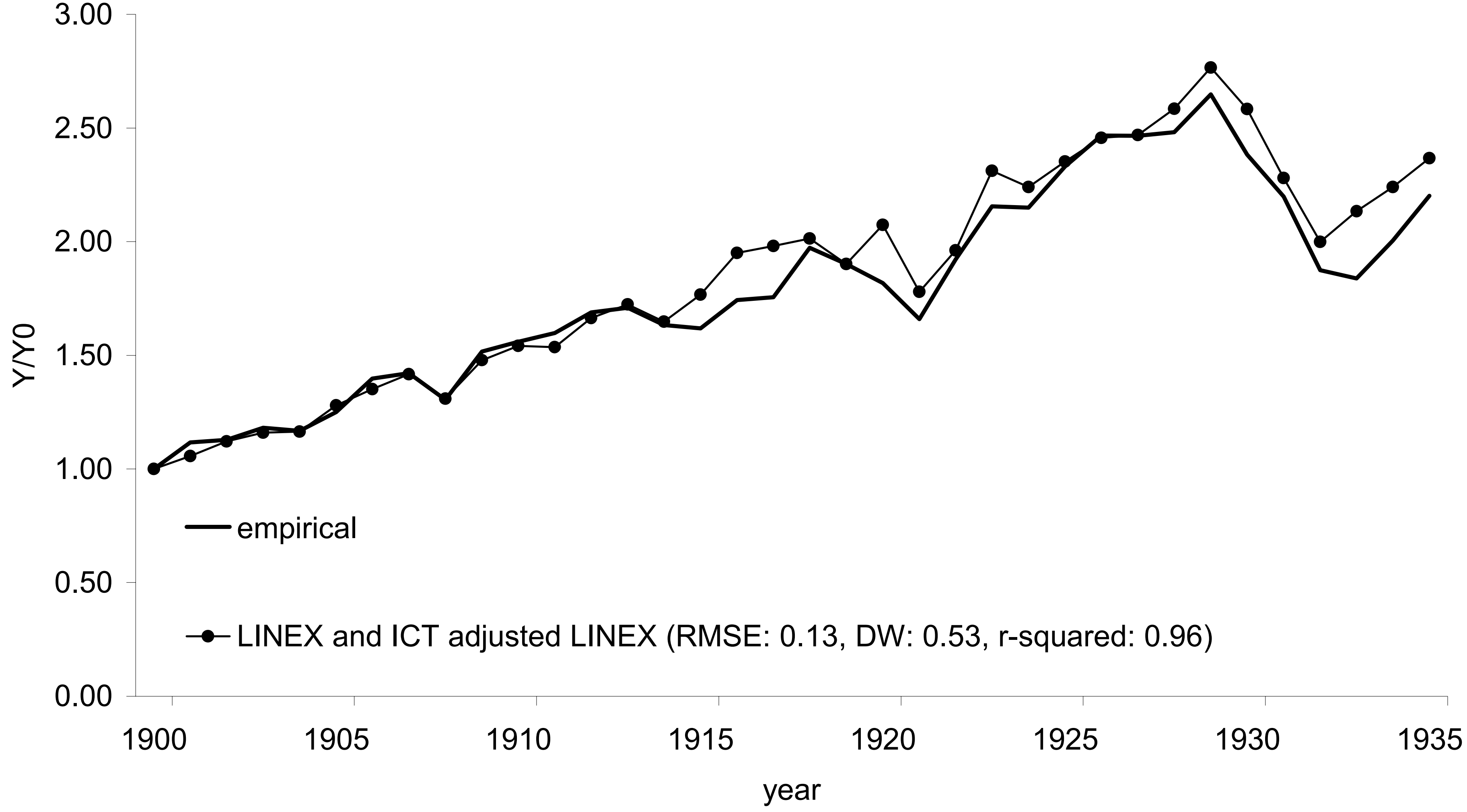


Figure 8. LINEX estimates of output, USA 1935-1970

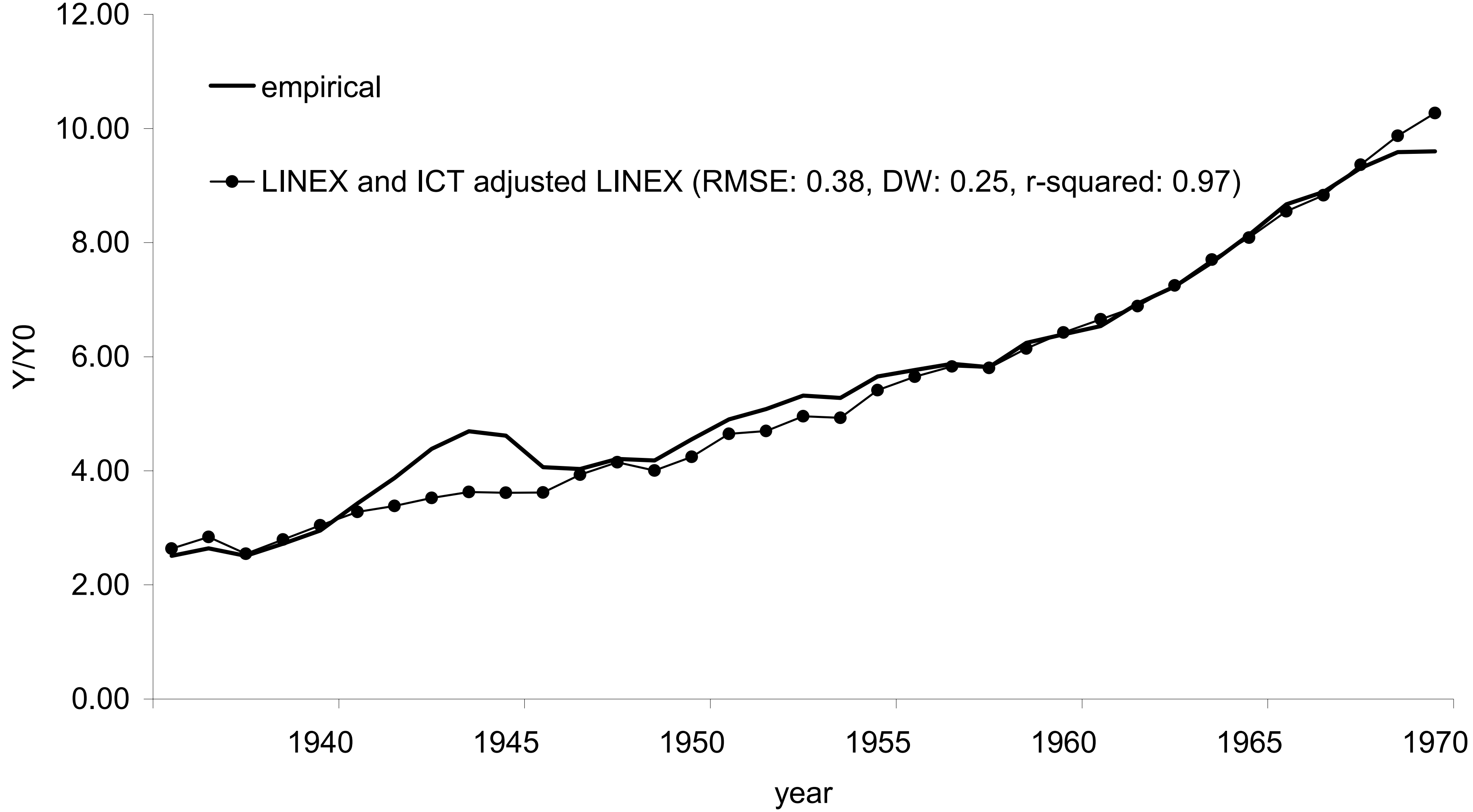


Figure 9. LINEX estimates of output, USA 1970-2000

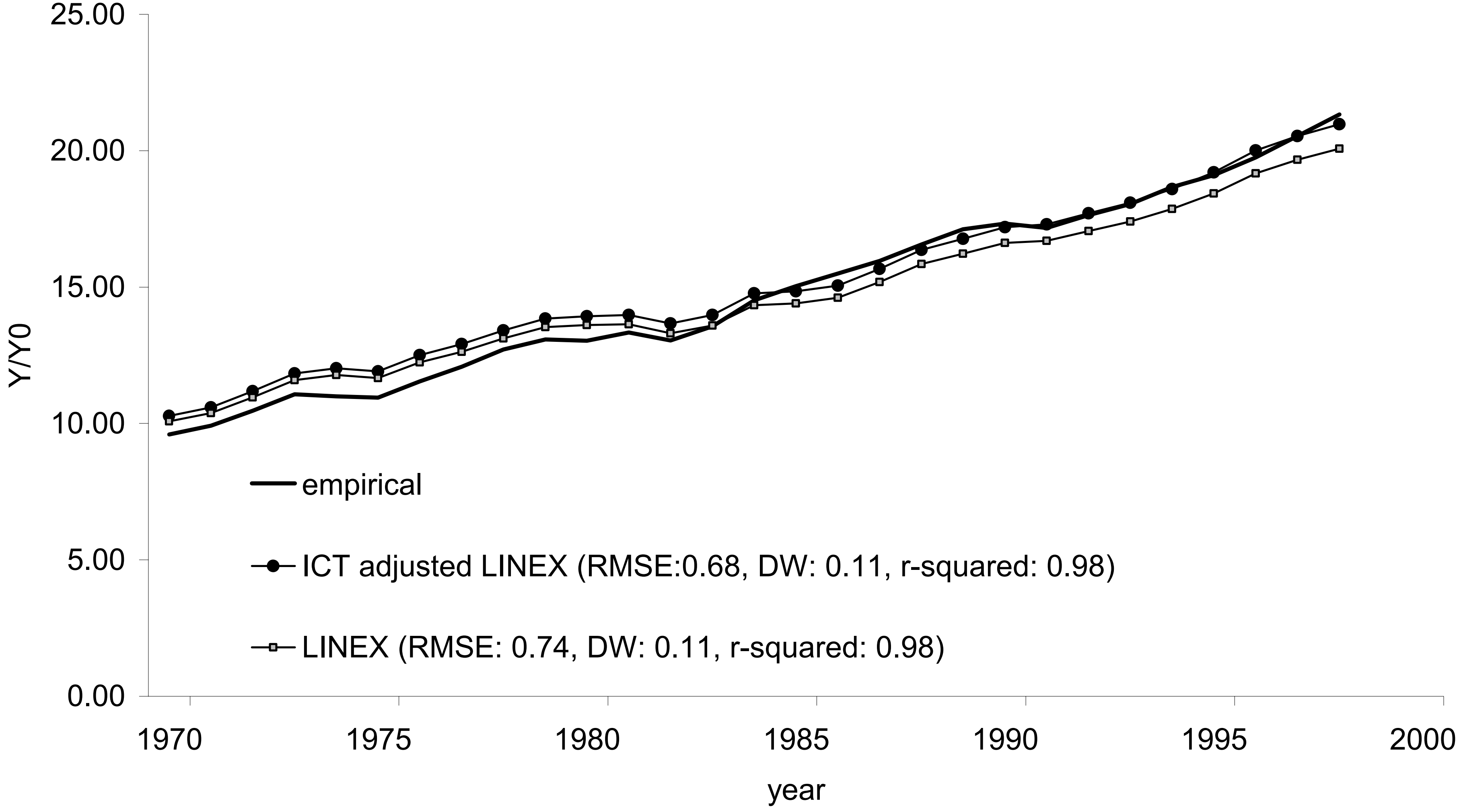


Figure B1. Inputs to the LINEX production function, USA 1900-2000